TRANSFER OF POWERFUL LASER RADIATION IN OPTICAL MEDIA: APPEARANCE OF "OPTICAL TURBULENCE"

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The passage of powerful laser radiation in a flat layer of diathermal medium with specularly reflecting and diffusely emitting boundaries is studied. The conditions under which "optical turbulence" arises are examined. It is shown that the optical characteristics of the boundaries of the flat layer are bifurcational parameters.

The development of laser technology has stimulated the investigation of different nonlinear optical phenomena arising in media through which laser radiation passes. In most works on nonlinear optics, however, the effect of the boundary surfaces of the optical media on the manifestation of various effects associated with the transfer of the laser radiation is not adequately taken into account.

In this paper it is shown that when powerful laser radiation passes through optical media the properties of the boundary surfaces can significantly affect the intensity of the light flux and in some cases they can result in the appearance of "optical turbulence" [1].

1. We shall study the passage of laser radiation through a flat layer of a diathermal medium with opaque diffusely emitting and specularly reflecting boundaries (Fig. 1). Then the nonstationary radiation transfer problem can be written in the following form:

$$\frac{1}{c}\frac{\partial J_{1,\lambda}}{\partial t}+\frac{\partial J_{1,\lambda}}{\partial x}=0,\quad \frac{1}{c}\frac{\partial J_{2,\lambda}}{\partial t}-\frac{\partial J_{2,\lambda}}{\partial x}=0, \tag{1}$$

where $(x, t) \in \Pi = [0, \ell] \times [0, +\infty)$, $\ell > 0$, with the boundary conditions

$$\begin{aligned} J_{1,\lambda} &= \frac{\varepsilon_{1,\lambda}}{\pi} n_\lambda^2 J_0(\lambda, T) + R_{1,\lambda} J_{2,\lambda}|_{x=0,\, t\geqslant 0},\\ J_{2,\lambda} &= \frac{\varepsilon_{2,\lambda}}{\pi} n_\lambda^2 J_0(\lambda, T) + R_{2,\lambda} J_{1,\lambda}|_{x=l,\, t\geqslant 0} \end{aligned} \tag{2}$$

and initial conditions

$$J_{1,\lambda}(x, 0) = J_p(x),\quad J_{2,\lambda}(x, 0) \equiv 0,\quad 0 \leqslant x \leqslant l. \tag{3}$$

This formulation of the problem is valid in the case of the passage of comparatively weak radiation fluxes in an optical medium. If, however, the passage of powerful monochromatic radiation in a medium with specularly reflecting and diffusely emitting boundaries is studied, then the boundary conditions can become substantially nonlinear, since the optical properties of the boundaries will depend on the intensity of the radiation incident on the surface. Then, for the transfer of powerful laser radiation in a diathermal medium, the boundary condition (2) can be generalized as follows:

$$\begin{aligned} J_{1,\lambda} &= \frac{\varepsilon_{1,\lambda}}{\pi} n_\lambda^2 J_0(\lambda, T) + f_1(J_{2,\lambda})|_{x=0,\, t\geqslant 0},\\ J_{2,\lambda} &= \frac{\varepsilon_{2,\lambda}}{\pi} n_\lambda^2 J_0(\lambda, T) + f_2(J_{1,\lambda})|_{x=l,\, t\geqslant 0} \end{aligned} \tag{2'}$$

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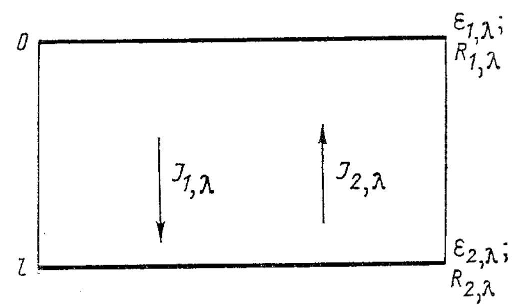

Fig. 1. Transfer of radiation in a flat diathermal medium.

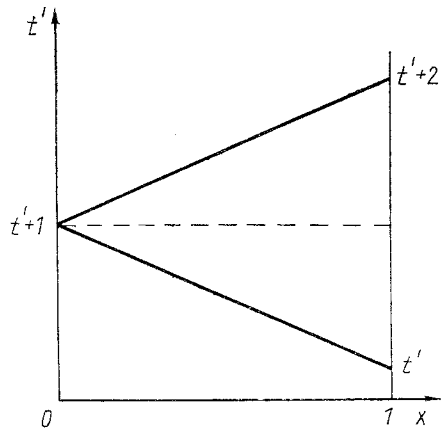

Fig. 2

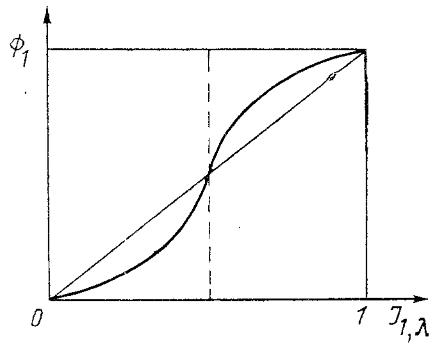

Fig. 3

Fig. 2. The geometric scheme of reduction to a difference equation.

Fig. 3. Form of the reflection function at the boundary of the region.

or

$$
\begin{aligned}
J_{1,\lambda} &= \Phi_{1,\mu_1}(J_{2,\lambda}) \quad \text{at} \quad x=0,\\
J_{2,\lambda} &= \Phi_{2,\mu_2}(J_{1,\lambda}) \quad \text{at} \quad x=1
\end{aligned} \tag{2''}
$$

if $\varepsilon_{1,\lambda}$, $\varepsilon_{2,\lambda}$, n_λ and T are constants. Here, evidently

$$\mu_i = \frac{\varepsilon_{i,\lambda}}{\pi} n_\lambda^2 J_0(\lambda, T), \; i = 1,\, 2.$$

We denote $t' = ct$ and, without loss of generality, we set $\ell = 1$.* Then the solution of the problem (1), (2"), and (3) can be represented in the form

$$J_{i;\lambda}(x, t') = Y_\lambda(t' + (-1)^i x), \; i = 1,\, 2,$$

where $Y_\lambda(t')$ is the solution of the difference equation

$$Y_\lambda(t'+2) = \Phi_{(\cdot)}(Y_\lambda(t')), \; t' \in [-1, \infty), \tag{4}$$

with the initial condition

$$Y_\lambda(t')|_{[-1,1)} = h(t') \overset{\text{def}}{=} \begin{cases} J_p(-t'), & t' \in [-1, 0), \\ 0, & t' \in [0, 1); \end{cases} \tag{5}$$

here $\Phi(\cdot) = \Phi_{1,\mu_1} \circ \Phi_{2,\mu_2}$ is the superposition of the corresponding functions.

*From the presentation given below it will be easy to see how the corresponding results change for arbitrary $\ell > 0$.

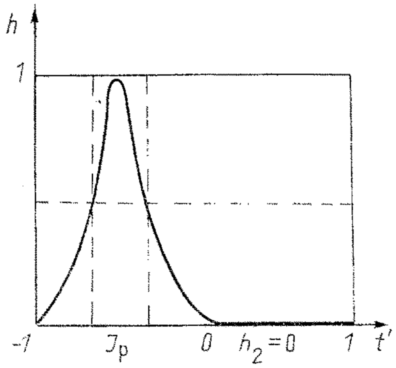

Fig. 4

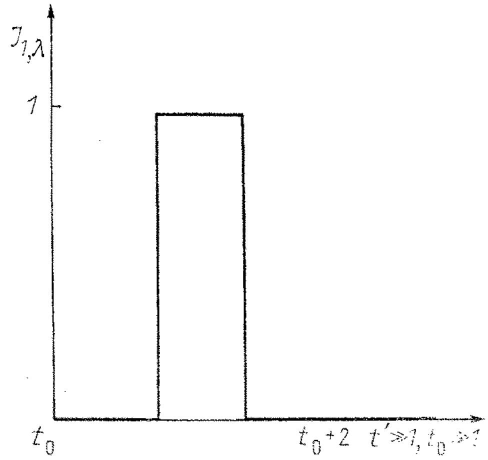

Fig. 5

Fig. 4. Initial distribution of the incident radiation.

Fig. 5. Form of the limiting distribution of the radiation.

Thus the starting problem has been reduced to a difference equation (4) with continuous time and the initial condition (5). The method of reduction, whence, in particular, the relation (5) follows, is studied in detail in [2]. In [2] the conditions which the functions $\Phi_{(\cdot)}$ and h must satisfy are indicated and a theorem on the asymptotic behavior (in the limit $t \to \infty$) of the solutions of the problem (1), (2"), and (3) is formulated.

Here we note that the behavior of the solutions depends on the topological properties of the mapping $\Phi_{(\cdot)}$, in particular, on the structure of the set of stationary points of $\Phi_{(\cdot)}$, i.e., points $\xi \in R^1$ such that $\xi = \Phi_{(\cdot)}(\xi)$, or some iterations

$$\xi_n = \Phi_{(\cdot)}(\xi_{n-1}) = \Phi_{(\cdot)} \circ \Phi_{(\cdot)}(\xi_{n-2}) = \ldots = \Phi^n_{(\cdot)}(\xi_1) = \xi_1,$$

which are called cycles of period n.

Thus, according to [2], piecewise—smooth asymptotically periodic solutions with finite, infinite, and even uncountable (of the type "Cantor comb") set of points of discontinuities in a period are typical for Eq. (4) (and therefore for the starting problem). For example, the solution can approch in the limit $\tau \to \infty$ a period-2 function of τ' (or period-2/c function of the variable t).

2. We shall study the specific example when the boundary conditions have the form (2'). following the method of characteristics (Fig. 2), we write the chain of relations

$$J_{1,\lambda}(1,\ t'+2) = J_{1,\lambda}(0,\ t'+1) = \frac{\varepsilon_{1,\lambda}}{\pi} n_\lambda^2 J_0(\lambda,\ T) +$$
$$+ f_1(J_{2,\lambda}(0,\ t'+1)) = \frac{\varepsilon_{1,\lambda}}{\pi} n_\lambda^2 J_0(\lambda,\ T) + f_1(J_{2,\lambda}(1,\ t')) =$$
$$= \frac{\varepsilon_{1,\lambda}}{\pi} n_\lambda^2 J_0(\lambda,\ T) + f_1\left(\frac{\varepsilon_{2,\lambda}}{\pi} n_\lambda^2 J_0(\lambda,\ T\right) + f_2(J_{1,\lambda}(1,\ t')).$$

We set in the last equality $f_1 \overset{\text{def}}{=} \mathrm{Id},\ f_2 = f$, where Id is the identity mapping. Then

$$J_{1,\lambda}(1,\ t'+2) = f(J_{1,\lambda}(1,\ t')) + \mu,\ t' \geqslant 0, \tag{6}$$

where

$$\mu = \frac{1}{\pi}(\varepsilon_{1,\lambda} + \varepsilon_{2,\lambda}) n_\lambda^2 J_0(\lambda,\ T).$$

Next, let $\varepsilon_{1,\lambda} = \varepsilon_{2,\lambda} = \varepsilon_\lambda/2$. Then

$$\mu = \frac{\varepsilon_\lambda}{\pi} n_\lambda^2 J_0(\lambda, T). \tag{7}$$

It is known [2] that the behavior of the solutions of the difference equation (6) is determined by the behavior of the trajectories of the one-dimensional dynamical system generated by the mapping

$$\varphi_\mu : J_{1,\lambda} \to f(J_{1,\lambda}) + \mu.$$

For definiteness, we shall examine the simplest example of a one-dimensional mapping φ_μ: $u \to (1/2)u^2 + \mu$, where $u \overset{\mathrm{def}}{=} J_{1,\lambda}$. For any $-4 \le \mu \le 0.5$ the mapping φ_μ has an invariant interval J_μ and the solutions generated by the initial functions $h = \{J_p, 0\}$, whose values belong to the interval J_μ, remain in J_μ for all $t' \ge 0$. Depending on the values of the parameter μ the solutions of this problem 1a) can be asymptotically stationary and 2a) they can approach asymptotically piecewise-constant functions, which are periodic as a function of t' (for each fixed $x \in (0, 1)$) and whose frequency of oscillation either remains constant in time or increases without bound in a power-law fashion.

In the case 1a) we have a solution of the relaxational type; in the case 2a) the solution is of a preturbulent type. In the first case the set of points of discontinuities of the limit function (points at which the derivative approaches infinity) is finite, while in the second case it is countable.

There also exists a set of parameters $\mu \in R^1$ for which the set of points of discontinuities of the limiting solution is uncountable (it is homeomorphic to Cantor's set [2]). The number of oscillations of the solution in any time interval $(t_0, t_0 + 2)$ increases exponentially as $t_0 \to \infty$. Such solutions are called solutions of the turbulent type (in the case at hand "optical turbulence").

Returning to the specific form of the parameter μ, determined by the relation (7), we can say that each parameter n_λ, ε_λ, and $T = \mathrm{const}$ is a bifurcational parameter in the above-indicated sense of the change in the qualitative behavior of the solutions. These same parameters are also bifurcational in the standard sense. For example, there exists a countable set of parameters $n_{\lambda,j}$, $j = \overline{0, \infty}$ at which the period and the frequency of oscillations of the function $J_{1,\lambda}$ (or $J_{2,\lambda}$) change. It is obvious that this is also true for the other parameters of the problem.

In typical situations the bifurcations of the solutions are accompanied by a change in periods in accordance with Sharkovskii's universal order [2]:

$$1\Delta 2\Delta 4\Delta \ldots \Delta 5\cdot 2\Delta 3\cdot 2\Delta \ldots \Delta 5\Delta 3.$$

The first chain of "inequalities" means that the solutions have period-doubling bifurcations well known from hydrodynamics [3]. Finally, the period doubling is characterized by a universal rate $\delta = 4.669\ldots$ and by a universal ratio of the amplitudes of the oscillations of the spectral intensity of the radiation $\alpha = 2.502$ [4].

For the problem at hand, other "phenomena," which in the last few years have been intensively studied in many works [4, 5], are also possible.

In conclusion we shall present a graph of the simplest limiting distribution of the radiatic component $J_{1,\lambda}$, when the initial pulse J_p has one oscillation and the mapping $\Phi(\cdot)$ is monotonic (Figs. 3-5).

NOTATION

Here $J_{1,\lambda}$ and $J_{2,\lambda}$ are the spectral intensities of the radiation; J_p is the intensity of the incident radiation; $J_0(\lambda, T)$ is Planck's function; c is the velocity of light; λ is the wavelength; $\varepsilon_{1,\lambda}$ and $\varepsilon_{2,\lambda}$ are the spectral emissivities of the boundary surfaces; n_λ is the spectral index of refraction; and, $R_{1,\lambda}$ and $R_{2,\lambda}$ are the spectral reflection coefficients of the boundaries.

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